Using covariates

$$\theta_{it} \mid \theta_{i0} \sim N(\theta_{i0}, \sigma^2)$$

$$\downarrow$$

① Suppose we have covariates available for **all** areas.

$$\cancel{(\theta_{i0} \mid \underset{\sim}{X}_{i0}) \sim N(X_{i0}\underset{\sim}{\beta}, \tau^2)}$$

$$\cancel{(\theta_{it} \mid \theta_{i0}, X_{it}) \sim N(\theta_{i0} + X_{it}\underset{\sim}{\beta},}$$

$$(\theta_{i0} \mid X_{i0}) \sim N(X_{i0}\underset{\sim}{\beta}, \tau^2)$$

$$(\theta_{it} - \theta_{i0} \mid X_{it} - X_{i0}) \sim N((X_{it} - X_{i0})\underset{\sim}{\beta}, \sigma^2)$$

Full model:

$$\hat{\theta}_{it} \mid \theta_{it} \sim N(\theta_{it}, \psi_{it})$$

$$\theta_{it} \mid \theta_{i0} \sim N(\theta_{i0} + (X_{it} - X_{i0})\underset{\sim}{\beta}, \sigma^2)$$

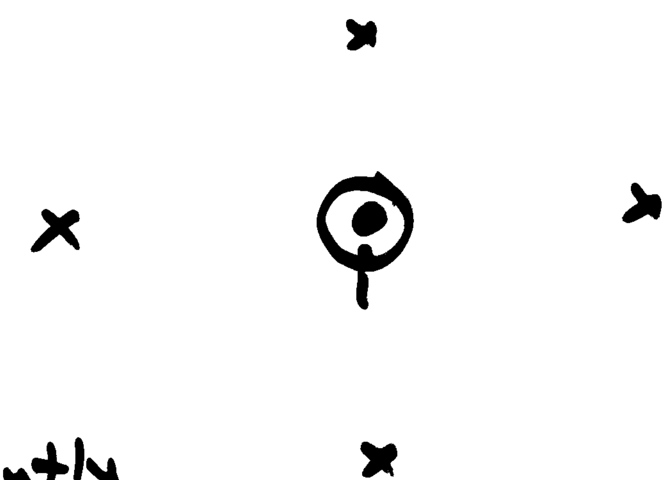
✓ $$\theta_{i0} \mid X_{i0} \sim N(X_{i0}\underset{\sim}{\beta}, \tau^2)$$

Shrinking factor is a function of $\boxed{\dfrac{\sigma^2}{\psi_{it}}}$

Spatial dependence

$$\hat{\theta}_{it} \mid \theta_{it} \sim N(\theta_{it}, \psi_{it})$$

$$\theta_{it} \mid \theta_{i0} \sim N\Big(\theta_{i0} + \rho \underbrace{\sum_{j \in N(i)} \theta_{jt}}_{\text{These are proxy for the covariates}}, \sigma^2\Big)$$



Equivalently

$$(\underset{\sim}{\theta}_t \mid \underset{\sim}{\theta}_0) \sim N(\underset{\sim}{\theta}_0, \underline{V})$$

V : Variance matrix

$$\mathrm{Cov}(\theta_{it}, \theta_{jt}) = \sigma^2 \cdot \rho^{d_{ij}}$$

General model

1. $\hat{\theta}_{it} \mid \theta_{it} \sim N(\theta_{it}, \psi_{it})$

2. $\underset{\sim}{\theta}_t \mid \underset{\sim}{\theta}_0 \sim N(\underset{\sim}{\theta}_0, V)$

or

$$\underset{\sim}{\theta}_t \mid \underset{\sim}{\theta}_0 \sim N(\underset{\sim}{\theta}_0 + (\underline{X}_t - \underline{X}_0)\underset{\sim}{\beta}, V)$$